Mathematical Games II

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Purpose

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- Understanding games with strategy.

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- Adopting symmetry strategy.

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Some suggestions

- Try for a smaller number.
- Look at the endgame.
- Work backwards.

Returning back ...

Problem.

There are two piles with 5 pebbles each. In each turn, a player removes some number of pebbles (at least 1) from only one pile. The player who empties the board wins.
What is the strategy to win? Who has the winning strategy, first player or the second?

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hint: We note that 2nd player always wins in case when there are two piles with 2 pebbles each. Same with the 3 pebbles case.

Can we generalize?

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Can we generalize?

Yes, whatever number of pieces the first player picks from one of the piles, the second player picks the same number of pieces from the second pile. This is called the symmetric strategy. As the game progresses, we come to two pebbles or the 3 pebbles case.

So, the 2nd player wins. The winning strategy is to go for the 'symmetric move'.

How can we use the symmetric strategy here?

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Two players take turns in placing bishops on the squares of the chessboard so that they cannot capture each other. The player who cannot place a bishop looses. Who has the winning strategy?

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Answer.2nd player has the winning strategy.

Solution.Consider the horizontal line after 4th row as the `symmetric' line. The 2nd player can place a bishop symmetrically along that line which is a safe position, thereby replying every move of the first player.

Open question

What happens if the chessboard is 7×7?

Another problem with symmetric flavour

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Two players take turns in placing dominoes on the squares of the chessboard so that they do not intersect. The player who cannot place a domino looses. Who has the winning strategy?

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Solution. Use the same strategy used for the bishop's problem. For a domino placed vertically on 4th and 5th row, there will be a symmetric horizontal equivalent.

Winning position - 1

Problem.

A rook(elephant) is initially placed on left-bottom. A player can move it any number of squares to right or vertically upwards. The player who moves the rook to right-top wins. Who has the winning strategy?

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Answer. 2nd player has the winning strategy.

Solution. For every hor/ver move done by the 1st player, make a corresponding ver/hor move to bring the rook on the diagonal connecting left-bottom and top-right.

Winning position - 2

Problem.

There are two piles with 10 and 11 stones. A move consists of discarding one of the piles and dividing the other pile into two non-empty piles. The player who cannot divide any further looses. Who has the winning strategy?

Winning position - 2

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There are two piles with 10 and 11 stones. A move consists of discarding one of the piles and dividing the other pile into two non-empty piles. The player who cannot divide any further looses. Who has the winning strategy?

Answer. 1'st player has the winning strategy.

Winning position - 2

Problem.

There are two piles with 10 and 11 stones. A move consists of discarding one of the piles and dividing the other pile into two non-empty piles. The player who cannot divide any further looses. Who has the winning strategy?

Answer. 1'st player has the winning strategy.

Solution. 1st player wins. He/she discards the odd pile and divide the even pile into two odd parts.

Winning position - 3: Watch the pattern

Problem.

There are 60 matches in a matchbox. A move consists of removing not more than half the number of matches present in the box. The player unable to do so looses. Who has the winning strategy?

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Answer. 2nd player has the winning strategy.

Solution. He/She is already in the winning position. The strategy would be to send the opponent into a loosing position.