Week 6: Misc Problems

http://bit.ly/trig2013

Srikanth K S

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4. If $\cos 5x = \cos^5 x + B\cos^3 x + C\cos x$. Find A, B and C.

Sol:

$$
\begin{aligned}
\cos 5x &= \cos(2x + 3x) \\
&= \cos 2x \cos 3x - \sin 2x \sin 3x \\
&= (2\cos^2 x - 1)(4\cos^3 x - 3\cos x) - (2\sin x \cos x)(3\sin x - 4\sin^3 x) \\
&= (8\cos^5 x - 6\cos^3 x - 4\cos^3 x + 3\cos x) - 2\cos x(3\sin^2 x - 4\sin^4 x) \\
&= (8\cos^5 x - 10\cos^3 x + 3\cos x) - 2\cos x(3(1 - \cos^2 x) - 4(1 - \cos^2 x)^2) \\
&= (8\cos^5 x - 10\cos^3 x + 3\cos x) - 2\cos x(3 - 3\cos^2 x - 4(1 + \cos^4 x - 2\cos^2 x)) \\
&= (8\cos^5 x - 10\cos^3 x + 3\cos x) - 2\cos x(-4\cos^4 x + 5\cos^2 x - 1) \\
&= (8\cos^5 x - 10\cos^3 x + 3\cos x) + (8\cos^5 x - 10\cos^3 x + 2\cos x) \\
&= 16\cos^5 x - 20\cos^3 x + 5\cos x
\end{aligned}
$$

5. If $2\tan a = 3\tan b$, Prove that

$$\tan(a - b) = \frac{\sin 2b}{5 - \cos 2b}$$

sol:

$$
\begin{aligned}
LHS &= \tan(a-b) \\
&= \frac{\tan a - \tan b}{1 + \tan a \tan b} \\
&= \frac{\frac{3}{2}\tan b - \tan b}{1 + \left(\frac{3}{2}\tan b\right)\tan b} \\
&= \frac{\tan b}{2 + 3\tan^2 b}
\end{aligned}
$$

$$
\begin{aligned}
RHS &= \frac{\sin 2b}{5 - \cos 2b} \\
&= \frac{\frac{2\tan b}{1+\tan^2 b}}{5 - \frac{1-\tan^2 b}{1+\tan^2 b}} \\
&= \frac{2\tan b}{5 + 5\tan^2 b - 1 + \tan^2 b} \\
&= \frac{\tan b}{2 + 3\tan^2 b}
\end{aligned}
$$

9. If $a+b+c = 180°$, prove that $\sin a + \sin b - \sin c = 4\cos\left(\frac{a}{2}\right)\cos\left(\frac{b}{2}\right)\cos\left(\frac{c}{2}\right)$

sol:

$$
\begin{aligned}
\sin a + \sin b - \sin c &= 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right) + 2\sin\left(\frac{c}{2}\right)\cos\left(\frac{c}{2}\right) \\
&= 2\cos\left(\frac{c}{2}\right)\cos\left(\frac{a-b}{2}\right) + 2\sin\left(\frac{c}{2}\right)\cos\left(\frac{c}{2}\right) \\
&= 2\cos\left(\frac{c}{2}\right)\left(\cos\left(\frac{a-b}{2}\right) + \sin\left(\frac{c}{2}\right)\right) \\
&= 2\cos\left(\frac{c}{2}\right)\left(\cos\left(\frac{a-b}{2}\right) + \cos\left(\frac{a+b}{2}\right)\right) \\
&= 2\cos\left(\frac{c}{2}\right)\left(2\cos\left(\frac{a}{2}\right)\cos\left(\frac{b}{2}\right)\right) \\
&= 4\cos\left(\frac{a}{2}\right)\cos\left(\frac{b}{2}\right)\cos\left(\frac{c}{2}\right)
\end{aligned}
$$

10. Assuming $\sin x \neq 0$ and $\cos x \neq 0$, prove that

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \geq 2$$

sol:

$$\begin{aligned}
\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \\
&= \frac{1}{\sin x \cos x} \\
&= \frac{2}{2 \sin x \cos x} \\
&= \frac{2}{\sin 2x} \\
&\geq 2 \qquad\qquad\qquad \text{since } \; -1 \leq \sin 2x \geq 1
\end{aligned}$$