

Week 29: Problems (Limit of a function / continuity II)

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Srikanth K S

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1 Problems

1. For $a > 0$,

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

2.

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} \cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) \right)$$

3. For $a > 0$,

$$\lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$$

4.

$$\lim_{x \rightarrow 0} \left(\frac{\log(2 + x) - \log(2 - x)}{x} \right)$$

5.

$$\lim_{x \rightarrow 0} \frac{1 - 2^x - 5^x + 10^x}{x \sin x}$$

6.

$$\lim_{x \rightarrow 1} \left(\tan \left(\frac{x\pi}{2} \right) \log \left(2 - \frac{1}{x} \right) \right)$$

7. Find a and b provided:

$$\lim_{x \rightarrow 0} (1 + ax + bx^2)^{\frac{1}{x}} = e^3$$

8. Let n be a positive integer. Find,

$$\lim_{x \rightarrow \frac{\pi}{2}} \left(1^{\sec^2 x} + 2^{\sec^2 x} + \dots + n^{\sec^2 x} \right)^{\cos^2 x}$$

9. Let $i(x)$ denote the integral part of a real number x . Find

$$\lim_{n \rightarrow \infty} \frac{i(x) + i(2x) + \dots + i(nx)}{n^2}$$

where x is some real number.

10. Let

$$\begin{aligned} a_1 &= 1 \\ a_2 &= 1 + a_1 \\ a_3 &= 1 + a_1 a_2 \\ &\dots \\ a_n &= 1 + a_1 a_2 \dots a_{n-1} \end{aligned}$$

Find

$$\sum_{n=1}^{\infty} \frac{1}{a_n}$$

11. Let

$$f(x) = \lim_{n \rightarrow \infty} \frac{1 - x^{2n}}{1 + x^{2n}}$$

Find the points of discontinuity of the function.

12. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function such that the $f(x) = x$ does not have a solution. Does $f(f(x)) = x$ have a real solution?
