Week 21: Problems

(Properties of triangles)

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1. Prove (napier's analogy): $\frac{b-c}{b+c}\cot\left(\frac{A}{2}\right) = \tan\left(\frac{B-C}{2}\right)$
2. Prove (Mollweide's rule): $\frac{b-c}{a}\cos\left(\frac{A}{2}\right) = \sin\left(\frac{B-C}{2}\right)$
3. Prove (a variant of Mollweide's): $\frac{b+c}{a}\sin\left(\frac{A}{2}\right) = \cos\left(\frac{B-C}{2}\right)$
4. Find $\sum_{cyclic}(a-b)\sin C$
5. Find $\sum_{cyclic}(a+b)\cos C$
6. Find $\sum_{cyclic} a\sin(B-C)$
7. If $(a+b-c)(b+c-a) = 3ac$, then find $\angle B$
8. If $\tan\frac{A}{2} = \frac{5}{6}, \tan\frac{C}{2} = \frac{2}{5}$. Then, prove that a, b, c are in a AP (arithmetic progression).
9. Find $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$ in terms of s where $s = \frac{a+b+c}{2}$
10. $\cot A + \cot B + \cot = \frac{a^2+b^2+c^2}{4\Delta}$ where Δ is the area of the triangle.

11. $8R^2 = a^2 + b^2 + c^2 \iff$ Triangle is right-angled.

12. If $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ac}$, find $\angle C$

13. In a $\triangle ABC$, $\angle ABC = 45^\circ$. The point D is on the line segment BC such that $\frac{|BD|}{|DC|} = \frac{1}{2}$. If $\angle DAB = 15^\circ$, find $\angle ACB$

14. In a $\triangle ABC$, BE and CF are medians. If $\angle CBE = 30^\circ$, $\angle BCF = 60^\circ$ and $a = 1$, find the area of the triangle ABC

15. Find the length of the angular bisector in terms of the sides and the bisected angle.

16. Prove that $(a + b + c)\left(\tan \frac{A}{2} + \tan \frac{B}{2}\right) = 2c \cot \frac{C}{2}$

17. a^2, b^2, c^2 are in an AP $\iff \cot A, \cot B, \cot C$ are in an AP.

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