Week 20: Solutions

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course webpage: http://bit.ly/trig2013

1.

$$R = \frac{b}{2\sin B} = \frac{2}{2\sin\frac{\pi}{6}} = 2$$

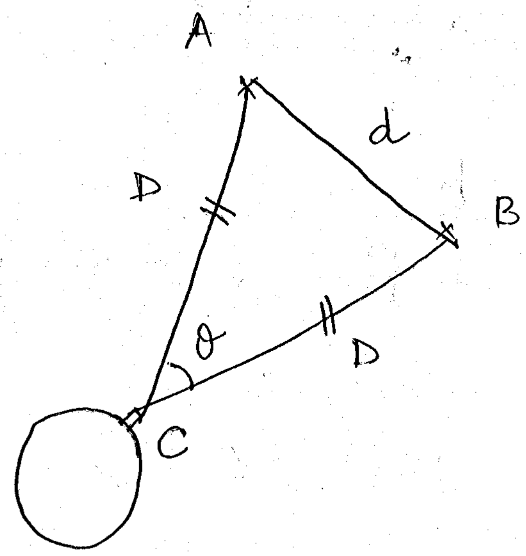
$$\angle C = 180^\circ - (\angle A + \angle B)$$
$$= \pi - \left(\frac{\pi}{6} + \frac{\pi}{12}\right) = \pi - \frac{3\pi}{12}$$
$$= \pi - \frac{\pi}{4}$$
$$= \frac{3\pi}{4}$$

$$c = 2R\sin C$$
$$= 2 \times 2 \times \sin\left(\frac{3\pi}{4}\right) = 2\sqrt{2}$$

$$a = 2R\sin A$$
$$= 2 \times 2 \times \sin\left(\frac{\pi}{12}\right)$$
$$= 4\ \cancel{\sin}\ \frac{\sqrt{3}-1}{2\sqrt{2}}$$
$$= \sqrt{2}(\sqrt{3}-1)$$

$\sin 15^\circ$
$= \sin(45-30)$
$= \sin 45 \cos 30 - \cos 45 \sin 30$
$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

2.



ΔABC is isoscles with $CA = CB$.

$$\Rightarrow \angle B = \frac{180^\circ - \theta^\circ}{2} = 90^\circ - \frac{\theta^\circ}{2}$$

Using Sine rule,

$$\frac{d}{\sin\theta} = \frac{D}{\sin\left(90 - \frac{\theta}{2}\right)}$$

$$\Rightarrow d = \frac{D\sin\theta}{\cos(\theta/2)}$$

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3. $$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

After simplification,

$$\Leftrightarrow a^2 + b^2 = c^2 + ab$$

$$\Leftrightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{1}{2} = \cos C \text{ (by cosine rule)}$$

$$\Leftrightarrow C = \frac{\pi}{3}$$

4.
$$\Delta = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} bc \cdot \frac{a}{2R} \quad \text{(by Sine rule)}$$

$$\boxed{\Delta = \frac{abc}{4R}}$$

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5. $$LHS = \frac{\cot A + \cot B + \cot C}{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}$$

$$= \frac{\sum_{cyc} \frac{\cos A}{\sin A}}{\sum_{cyc} \frac{\cos A/2}{\sin A/2}}$$

$$= \frac{\sum_{cyc} \frac{\frac{b^2+c^2-a^2}{2bc}}{\frac{2\Delta}{bc}}}{\sum_{cyc} \frac{\sqrt{\frac{s(s-a)}{bc}}}{\sqrt{\frac{(s-b)(s-c)}{bc}}}} \quad \text{(from problem 4)}$$

$$= \frac{\sum_{cyc} \frac{b^2+c^2-a^2}{4\Delta}}{\sum_{cyc} \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}}$$

$$= \frac{\frac{a^2+b^2+c^2}{4\Delta}}{\sqrt{s}\,\frac{s-a+s-b+s-c}{\sqrt{(s-a)(s-b)(s-c)}}}$$

$$= \frac{a^2+b^2+c^2}{4s(3s-(a+b+c))}$$ [Using Heron's formula]

$$= \frac{a^2+b^2+c^2}{2s(a+b+c)}$$

$$= \frac{a^2+b^2+c^2}{(a+b+c)^2}$$

6. Ans $= \dfrac{\sin(B+C)}{\sin(B-C)}$

$$= \frac{\sin^2(B+C)}{\sin(B+C)\sin(B-C)}$$

$$= \frac{\sin^2(180-A)}{\frac{1}{2}\left(\cos 2C - \cos 2B\right)}$$

$$= \frac{\sin^2 A}{\frac{1}{2}\left(1 - 2\sin^2 C - (1 - 2\sin^2 B)\right)}$$

$$= \frac{\sin^2 A}{\sin^2 B - \sin^2 C}$$

$$= \frac{(a/2R)^2}{(b/2R)^2 - (c/2R)^2} \quad \text{(by sine rule)}$$

$$= \frac{a^2}{b^2 - c^2}$$

(end)