

## **Week 20: Solutions**

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course webpage: <http://bit.ly/trig2013>

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1.

$$R = \frac{b}{2 \sin B} = \frac{2}{2 \sin \frac{\pi}{6}} = 2$$

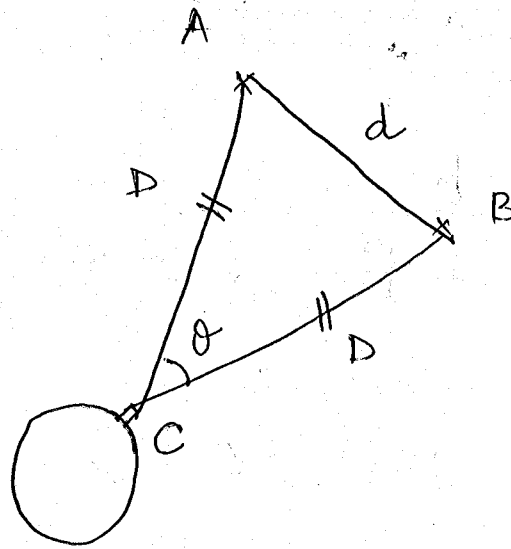
$$\begin{aligned} C &= 180^\circ - (A + B) \\ &= \pi - \left( \frac{\pi}{6} + \frac{\pi}{12} \right) = \pi - \frac{3\pi}{12} \\ &= \pi - \frac{\pi}{4} \\ &= \frac{3\pi}{4} \end{aligned}$$

$$\begin{aligned} c &= 2R \sin C \\ &= 2 \times 2 \times \sin \left( \frac{3\pi}{4} \right) = 2\sqrt{2} \end{aligned}$$

$$\begin{aligned} a &= 2R \sin A \\ &= 2 \times 2 \times \sin \left( \frac{\pi}{12} \right) \\ &= 4 \frac{\sqrt{3}-1}{2\sqrt{2}} \\ &= \sqrt{2}(\sqrt{3}-1) \end{aligned}$$

$$\begin{aligned} &\sin 15^\circ \\ &= \sin(45-30) \\ &= \sin 45 \cos 30 \\ &\quad - \cos 45 \sin 30 \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}} \end{aligned}$$

2.



$\triangle ABC$  is isosceles with  $CA = CB$ .

$$\Rightarrow \angle B = \frac{180^\circ - \theta^\circ}{2} = 90^\circ - \frac{\theta^\circ}{2}$$

Using Sine rule,

$$\frac{d}{\sin \theta} = \frac{D}{\sin \left( 90^\circ - \frac{\theta}{2} \right)}$$

$$\Rightarrow d = \frac{D \sin \theta}{\cos(\theta/2)}$$

3. 
$$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

After simplification,

$$\Leftrightarrow a^2 + b^2 = c^2 + ab$$

$$\Leftrightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2} = \cos C \quad (\text{by cosine rule})$$

4.

$$\Delta = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} bc \cdot \frac{a}{2R} \quad (\text{by Sine rule})$$

$$\Delta = \frac{abc}{4R}$$

$$5. \text{ LHS} = \cot A + \cot B + \cot C$$

$$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}$$

$$= \sum_{\text{cyc}} \frac{\cos A}{\sin A}$$

$$\sum_{\text{cyc}} \frac{\cos A/2}{\sin A/2}$$

$$= \sum_{\text{cyc}} \frac{b^2 + c^2 - a^2}{2bc} \cdot \frac{2\Delta}{bc}$$

(from problem 4)

$$\sum_{\text{cyc}} \frac{\sqrt{\frac{s(s-a)}{bc}}}{\sqrt{\frac{(s-b)(s-c)}{bc}}}$$

$$= \frac{\sum_{\text{cyc}} \frac{b^2+c^2-a^2}{4A}}{\sum_{\text{cyc}} \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$$

$$= \frac{\frac{a^2+b^2+c^2}{4A}}{\frac{\sqrt{s} \frac{s-a+s-b+s-c}{\sqrt{(s-a)(s-b)(s-c)}}}$$

$$= \frac{\frac{a^2+b^2+c^2}{4s(3s-(a+b+c))}}{\left[ \text{Using Heron's formula} \right]}$$

$$= \frac{\frac{a^2+b^2+c^2}{2s(a+b+c)}}$$

$$= \frac{a^2+b^2+c^2}{(a+b+c)^2}$$

$$\begin{aligned}
6. \quad \text{Ans} &= \frac{\sin(B+C)}{\sin(B-C)} \\
&= \frac{\sin^2(B+C)}{\sin(B+C)\sin(B-C)} \\
&= \frac{\sin^2(180-A)}{\frac{1}{2}(\cos 2C - \cos 2B)} \\
&= \frac{\sin^2 A}{\frac{1}{2}(1 - 2\sin^2 C - (1 - 2\sin^2 B))} \\
&= \frac{\sin^2 A}{\sin^2 B - \sin^2 C} \\
&= \frac{(a/2R)^2}{(b/2R)^2 - (c/2R)^2} \quad (\text{by sine rule}) \\
&= \frac{a^2}{b^2 - c^2}
\end{aligned}$$