# Lecture 19: Problems on ‘Inverse trigonometric functions’

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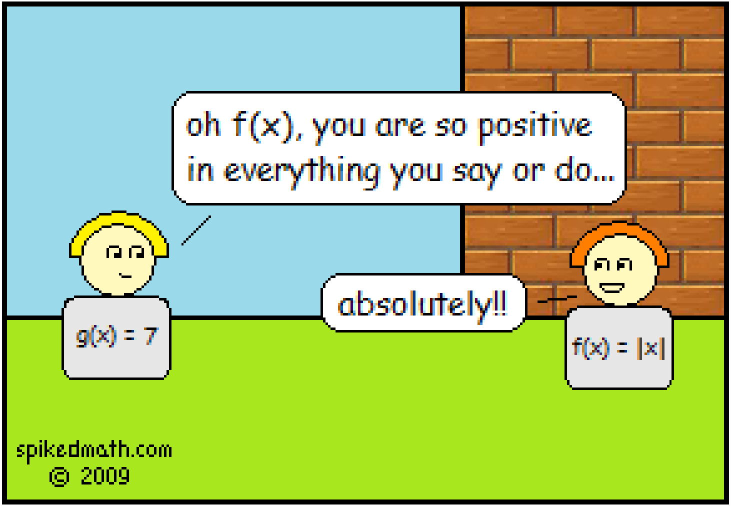
http://bit.ly/trig2013


oh f(x), you are so positive in everything you say or do...
absolutely!!
g(x) = 7
f(x) = |x|
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Problem.

The number of solutions of the equation

$$\cos^{-1}(x) + \cos^{-1}(2x) + \cos^{-1}(3x) = -\pi$$

is

(a) $0$

(b) 1

(c) 2

(d) None of these

**Problem.**

If

$$\sin^{-1}(\sin x) = \pi - x$$

Then,

(a) $x \in [0, \pi]$
(b) $x \in (0, \pi)$
(c) $x \in [\frac{\pi}{2}, \frac{3\pi}{2}]$
(d) None of these

Problem.

Provided $x, y, z$ are positive and $x^2 + y^2 + z^2 = r^2$, the value of

$$\tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{zx}{yr}\right) + \tan^{-1}\left(\frac{xy}{zr}\right)$$

is

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) $\pi$
(d) none of these

Problem.

Solve for $x$:

$$\tan^{-1}(x+1) + \tan^{-1}(x) + \tan^{-1}(x-1) = \tan^{-1}(3)$$

given $-\sqrt{2} < x < \frac{1}{3}$

(a) there is no solution other than $x = -1$

(b) there is a solution other than $x = -1$

(c) option (d) is impossible

(d) None of these

Problem.

$$\sum_{k=1}^{\infty} \tan^{-1}\left(\frac{2k}{k^4+k^2+2}\right)$$

equals to

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) None of these

Problem.

$$\sum_{n=1}^{\infty} \sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)$$

equals to

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) None of these

Problem.

$$\sin^{-1}(x) + \sin^{-1}(y) + \sin^{-1}(z) = \pi$$

If, $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = k(xyz)$, then $k$ is

(a) 1

(b) 2

(c) 3

(d) None of these

**Problem.**

The interval where $\sin^{-1}(x) < \cos^{-1}(x)$ is

(a) $\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

(b) $0 < x < \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}} < x < 1$

(d) None of these

Problem.

$$\left(\tan^{-1}(x)\right)^2 + \left(\cot^{-1}(x)\right)^2 = \frac{5\pi^2}{8}$$

(a) has exactly one solution
(b) has exactly two solutions
(c) has no solutions
(d) None of these

Problem.

If

$$\lfloor \sin^{-1}\left(\tan^{-1}(x)\right) \rfloor = 1$$

Then,

(a) $x \in [0, \tan(\pi - 1)]$

(b) $x \in [-\infty, \sin^{-1}(1)]$

(c) $x \in [\tan(\sin 1), \infty)$

(d) None of these

**Problem.**

If $p > q > 0$ and $pr < -1 < qr$, then

$$\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right)$$

equals to

(a) $0$

(b) $\pi$

(c) $\frac{3\pi}{2}$

(d) None of these

Problem.

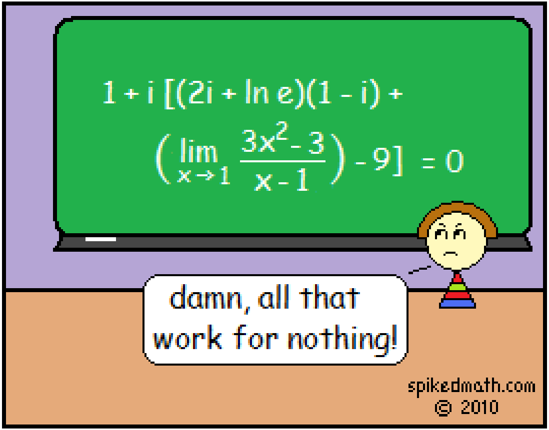
Let $\tan^{-1}(x), \tan^{-1}(y), \tan^{-1}(z)$ be in an AP(arithmetic progression) and $x, y, z$ be in an AP (a different AP) with $y$ not being either $0, 1$ or $-1$ and $y^2 < 1, xz < 1$. Then,

(a) x,y,z are in GP(geometric progression)

(b) $xy + yz + zx = 0$

(c) $x = y = z$

(d) None of these


$1 + i\,[(2i + \ln e)(1 - i) + \left(\lim_{x\to 1} \frac{3x^2 - 3}{x - 1}\right) - 9] = 0$
damn, all that work for nothing!
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