Week 18: Problems in 'inverse trigonometric functions'

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1. Find the value of $\sin^{-1}(\sin 5)$
2. Find the domain of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$
3. If $0 < x < 1$, find the value of $\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
4. Let $a = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ and $b = 2\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ where $0 < x < 1$. Then, $a + b$ equals to ...
5. If $\cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = \pi$, find the value of $x^2 + y^2 + z^2 + 2xyz$
6. If $0 \leq x \leq \frac{1}{2}$, find the value of $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right) - \sin^{-1}(x)\right)$
7. Simplify:
$$\sum_{k=1}^{n} \tan^{-1}\left(\frac{1}{1 + n(n+1)}\right)$$

8. Let $x > 0, y > 0, z > 0$. Simplify:

$$\sum_{\text{cyclic}} \tan^{-1}\left(\sqrt{\frac{x(x+y+z)}{yz}}\right)$$

9. Evaluate $\cos\left(2\cos^{-1}(x) + \sin^{-1}(x)\right)$ when $x = \frac{1}{5}$

10. Let the triangle be right-angled at vertex A, with B and C being other vertices. Its a usual practice to name the sides opposite to the vertices A, B, C as a, b, c respectively. Find the value of

$$\tan^{-1}\left(\frac{b}{c+a}\right) + \tan^{-1}\left(\frac{c}{a+b}\right)$$

11. <u>Statement 1:</u> If $x > y > z > 0$,

$$\sum_{\text{cyclic}} \cot^{-1}\left(\frac{1+xy}{x-y}\right) = \pi$$

<u>Statement 2:</u>

$$\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}(x) & \text{if } x > 0 \\ -\pi + \cot^{-1}(x) & \text{if } x < 0 \end{cases}$$

Are the statements true? Does 1 depend on 2?

12. Let x, y, z be in GP(all x,y,z are positive) with $xz < 1$ and $\mid y \mid \neq 1$ and $\tan^{-1}(x), \tan^{-1}(y), \tan^{-1}(z)$ are in AP. Prove that, $x = y = z$

13. If $\cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = 3\pi$. Then $xy + yz + zx$ equals to ...

14. Let $0 \leq p, q < 1$ and

$$\sin^{-1}\left(\frac{2p}{1+p^2}\right) - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

A possibility for x is ...

15. The number of values of x satisfying the equation $\sec^{-1}(x) + \sin^{-1}(x) = \frac{\pi}{2}$ is ...