

week14-15:Problems related to 'trigonometric equations and inequalities'

<http://bit.ly/trig2013>

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1 Problems

1.1 Problem set 1 (introductory)

Solve for x in the following equations:

1. $\cos 2x = \cos x$
2. $1 + 2 \sin^2 x = 3$
3. $\sin x + \sin 2x + \sin 3x = 0$
4. $\tan x + \cot x = 2 \csc x$
5. $\sec x + \csc x = 2\sqrt{2}$
6. $\cos x = x + \frac{1}{x}$

7. $\cos 3x = \cos^3 x$
8. $\sin 5x = \sin 3x$
9. $\sin 5x + \sin 3x = \cos 2x$
10. $\sin(m+n)x + \sin(m-n)x = \sin mx$

1.2 Problem set 2

1. The number of solutions for the pair of equations

$$\sin\left(\frac{x+y}{2}\right) = 0$$

and

$$|x| + |y| = 1$$

in $[0, 2\pi]$

2. Find the values for x in $[0, 2\pi]$ such that $\sin 2x < \sin x$
3. If $\cos(a-b) = -1$, then
 - a is not less than b
 - $\sin a + \sin b = 0$ and $\cos a + \cos b = 0$
 - Both a and b are positive
 - $\sin a + \sin b = 0$ but $\cos a + \cos b = 0$ need not be true.
4. If the equation $1 + \sin^2(kx) = \cos x$ has a non-zero solution for x , then x must be
 - an integer
 - an irrational
 - rational
 - any real number
5. The number of roots between 0 and π for the equation $2\sin^2 x + 12 = 3\sin x$ is
...

6. Find general solutions for $\tan x + \cot x = 4$
7. The number of solutions of $2\sin x + 3\cos x = 4$ is When does the equation $a\sin x + b\cos x = c$ have a solution?
8. If $\sin x(\sin x + \cos x) = k$ has a solution. Then, prove that $\frac{1-\sqrt{2}}{2} \leq k \leq \frac{1+\sqrt{2}}{2}$
9. Find the number of solutions to the equation $2x = 3\pi(1 - \cos x)$
10. Find the number of solutions to the equation $\sin 7x - \sin x = \sin 3x$ in $[0, \pi]$
11. If $\sin x = a$ has one solution in $(0, 7\pi/3)$. Then, $a = \dots$
12. Find all $x \in [-\pi, \pi]$ such that $|\tan x| \leq 1$
13. Let $r > 0, -\pi \leq x < \pi$ and $r\sin x = 3, r = 4(1 + \sin x)$. Find all (r, x) pairs.
14. Let $0 \leq x \leq 3\pi, 0 \leq y \leq 3\pi$ with $\cos x \sin y = 1$. Find the number of ordered pair solutions to the equations.