Lecture 13: Problems on functions

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Problem. The minimum value of the function $f : (0, \infty) \to \mathbb{R}$ such that $f(x) = x + \frac{1}{x}$ is

(a) 1

(b) 2

(c) There is no maximum value

(d) None of these

Problem. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that

$$f(x) = \begin{cases} 0 & \text{,if x is irrational} \\ 1 & \text{,if x is rational} \end{cases}$$

Then, the function f is

(a) periodic with a period

(b) not periodic

(c) periodic but without a period

(d) period depends on x

Problem. Let $f : \mathbb{R} \to \mathbb{R}$, such that

$$f(x) = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$

The image (range) of the function is,

(a) $[1/7, 7]$

(b) $[-1/7, 7]$

(c) $[-7, 7]$

(d) $(-\infty, 1/7) \cup (7, \infty)$

Problem. Suppose $f : \mathbb{R} \to \mathbb{R}$ such that $2f(x) + 3f(-x) = 11 \quad , \forall x \in \mathbb{R}$. Then, f is

(a) there does not exist such a f

(b) f is a constant function

(c) f is bijective

(d) None of these

Problem. $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^3 - x$ is

(a) ~~inj~~, sur

(b) inj, ~~sur~~

(c) ~~inj~~,~~sur~~

(d) inj, sur

(more than one option can be correct)

Problem. Let $f : \mathbb{R} \to \mathbb{R}$ be an strictly increasing function. Then, f is

(a) f is surjective

(b) f is injective

(c) f cannot miss exactly one point in $\mathbb{R}$

(d) f cannot miss exactly two points in $\mathbb{R}$

Problem. Let a, b, c be fixed positive numbers such that $f : \mathbb{N} \to \mathbb{R}, f(n) = \frac{na}{b+nc}$ such that

(a) f is increasing

(b) f is decreasing

(c) f increases first and then decreases

(d) None of these

Let $f : \mathbb{R} \to \mathbb{R}, f(a) + f(b) = f(a)f(b)$. Then,

(a) The image(range) cannot have more than one element.

(b) The image(range) cannot have more than two elements.

(c) The image set can have infinitely many elements.

(d) We cannot say anything about the image set.

Problem. The symmetric difference of two sets is defined as
$A * B := (A \setminus B) \cup (B \setminus A)$.

Let R be a relation on the set $2^{\mathbb{N}} :=$ Set of all subsets of $\mathbb{N}$. We have ${}_{A}R_{B}$ if $A * B \neq \emptyset$.

(a) ~~reflexive~~, symmetric ,transitive

(b) reflexive, symmetric ,~~transitive~~

(c) ~~reflexive~~, ~~symmetric~~ ,~~transitive~~

(d) ~~reflexive~~, symmetric ,~~transitive~~

Problem. Pick the function that does not belong to the category. (observe their period)

$$1.\frac{1+\sin x}{\cos x(1+\csc x)} \quad 2.\,|\sin|x|| \quad 3.\frac{1}{1-\cos 2x} \quad 4.\,x+\cos x$$

(a) 1

(b) 2

(c) 3

(d) 4

Problem. Lets define $| f | := | f(x) |$ and $max(f, g)(x) := max(f(x), g(x))$ and $min(f, g)(x) := min(f(x), g(x))$. Then, $max(f, g)(x)$ in terms of $| f |$ and $| g |$ is $\ldots\ldots$

Problem. Let $f : \mathbb{Q} \to \mathbb{R}$ such that

$$f(x + y) = f(x) + f(y)$$

Then,

(a) f is a non-linear function

(b) $f(x) \equiv 0$

(c) $f(x) = cx$ for some $c \in \mathbb{R}$

(d) None of these

Problem. Let f be a polynomial in x of degree greater than 0. Which of the statements is true?

(a) $f(a) = 0 \Leftrightarrow (x - a)$ is a factor of $f(x)$

(b) $f(a) = 0 \Rightarrow (x - a)$ is a factor of $f(x)$

(c) $f(a) = 0 \Leftarrow (x - a)$ is a factor of $f(x)$

(d) $f(a) = 0 \not\Leftrightarrow (x - a)$ is a factor of $f(x)$

Problem. Let $I(x) = x \quad \forall\, x$. If $fog(x) = gof(x) = g(x) \quad \forall\, g(x)$. Then,

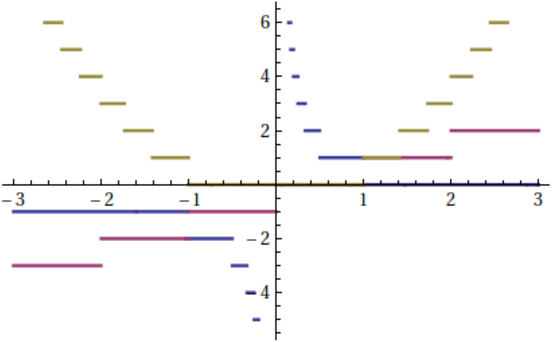
(a) $f = I$

(b) $f = I$ is just a possibility, f can be something else too.

(c) There is no such f

(d) None of these

Problem. Which of these plots is the correct depiction of $\lfloor \frac{1}{x} \rfloor$?



(a) blue

(b) green

(c) violet

(d) None of these

Problem. (last) A function F is said to be <u>increasing</u> if $F(x) \geq F(y)$ whenever $x > y$ and <u>strictly increasing</u> if $F(x) > F(y)$ whenever $x > y$.

Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous (we can draw the graph of the function without lifting the pen) and f is non-negative when x is positive and f is non-positive when x is negative. Then, which of these is not possible?

(a) f is increasing

(b) f is strictly increasing

(c) f is strictly decreasing

(d) f is decreasing