(Week 11) Class Handout: Periodic functions and their period

http://bit.ly/trig2013

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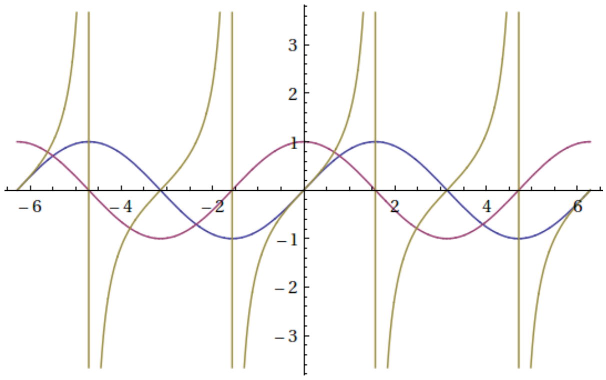
The points here are sketchy. You may have to fill in a lot of details.

1. A function $f : \mathbb{R} \to \mathbb{R}$ is said to be periodic if there exists some $p > 0$ such that $f(x + p) = f(x) \quad \forall x \in \mathbb{R}$
2. The smallest positive number p such that $f(x + p) = f(x)$ holds is called the period the function p.
3. The period of $\sin x, \cos x, \tan x$ are $2\pi, 2\pi, \pi$ respectively. (plot the graphs and observe)



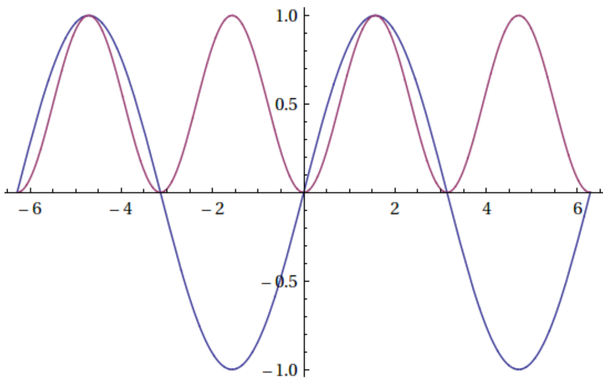
4. $\sin 2x$ has period 2π (observe it from the plot). In general, $\sin(kx)$ or $\cos(kx)$ have period $\frac{2\pi}{k}$, where k is some non-zero real number.

5. Note that if $f(x)$ is periodic, then $k_1 + k_2 f(x)$ is also periodic with the same period. (where k_1 and k_2 are some real constants.)

6. $\sin^2 x$ has period π.

   Argument 1: $\sin^2 x = \frac{1-\cos 2x}{2}$. From (4), the period of $\sin^2 x$ is π.

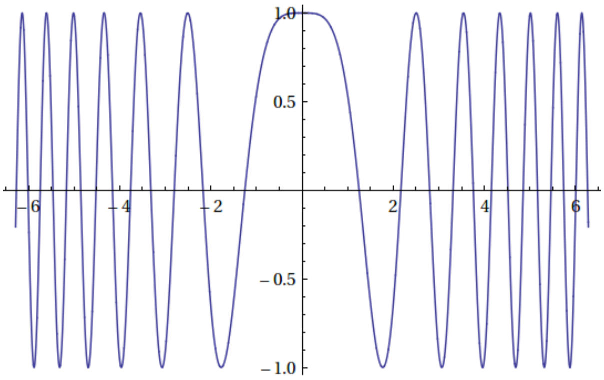
   Argument 2: Observe the plot.



7. We generalise on the same lines to prove that $\sin^{2k} x$ for some integer k has the period π. Otherwise, the function $\sin^k x$ has the period 2π.

8. If $h(x) = fog(x)$ exists, then $h(x)$ is periodic if $g(x)$ is periodic. (prove it!)

9. $\cos(x^2)$ is not periodic as the frequency is not constant.



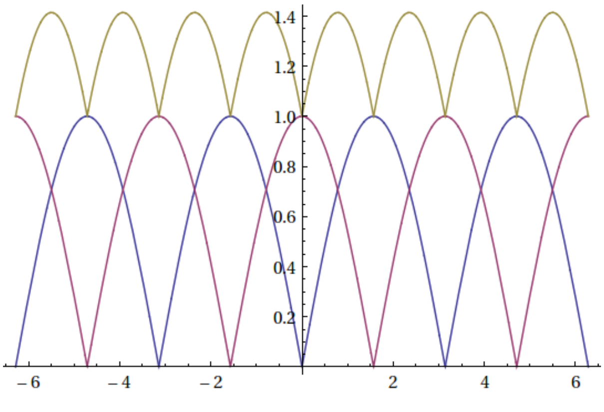
In general, sin or cos of a non-linear function is usually not periodic. Further, if the inner function (x^2) is not periodic, the composite function may fail to be periodic even if the outer function ($\cos x$) is periodic.

10. <u>LCM method:</u> Let f_1 and f_2 be periodic with periods p_1 and p_2 respectively. Then, $f_1 + f_2$ is periodic with period $\leq LCM(p_1, p_2)$

11. LCM method may fail when:

    (a) f_1 and f_2 have same periods.

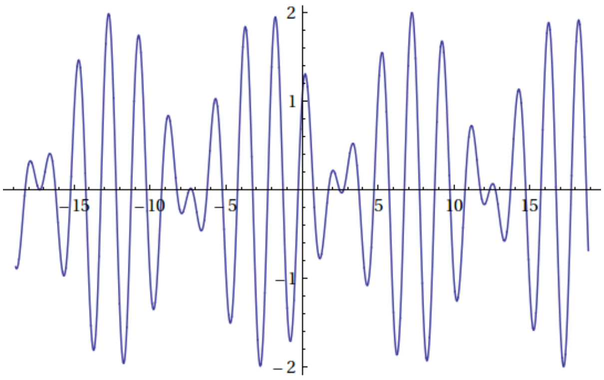
    example: $|\sin x| + |\cos x|$ seems to have the period π via LCM method. But the actual period is $\frac{\pi}{2}$. (prove it analytically!)



    (b) p_1 and p_2 are irrationals of different type.

    In this case, we cannot take LCM and the function is not periodic.

    example: $\sin(2\sqrt{2}x) + \sin 2\sqrt{3}x$



(look carefully, the function is not periodic)

12. Problems to find the period. First argue that the function is periodic qualitatively and then proceed to find the period.

    (a) $|\sin x|$

    (b) $2\cos\left(\frac{x+\pi}{4}\right)$

    (c) $\sin x + \cos x + \tan x$

    (d) $\sin(\cos(x))$

    (e) $\sin(\sqrt{x})$

    (d) $\lfloor \sin x \rfloor$

    (e) $\tan\left\lfloor \frac{x}{2} \right\rfloor$

    (f) $\sin x \cos^2 x$