# Lecture 10: A review quiz of first six weeks of trigonometry

Srikanth K S

http://bit.ly/trig2013

# Suggestions

Solving competitive questions is a different ball game compared to solving simple exercises. You are expected to have some insight in the subject matter and problem solving skills.

- Study all the basic theory properly. Spend some time with each formula, looking at how it used in a solved problem and think about how it can be used.
- Understand standard problem solving techniques (like checking the discriminant of a quadratic equation whenever required)
- Try different approaches. Do not give up. Initially, do not worry about the time you take to solve a problem. It is important that you get it right. Once you have the confidence that you can solve problems, start solving problems under a time limit.
- Maintain a book or dairy of important ideas. Write in detail: how you approached something, what your approach should have been. Have a list of things to improve upon, and mark your progress.

- Spend at least an hour or two a day with complete concentration while learning something or solving problem. During that time stay away from phone, facebook etc.. I prefer to have headphones on with mild music to cut off any ambient noise.
- Solving problem in a group is usually counter-productive. It helps to share your thoughts with a fellow student / teacher only after you have tried your own approach.
- Pay attention to detail. Sometimes, I may be wrong. A book may have a mistake. Do not accept an argument unless you are convinced about it.
- Most ideas tagged ``shortcuts'' do not work always, be careful about them. It is better to develop good techniques as you go on, instead of searching for ~~shortcuts~~ everywhere.
- Eat well, sleep properly, have fun and enjoy your life. Leading a disciplined life makes you better in whatever you do ☺☺☺

Problem. Let $A = \sin^8 x + \cos^8 x \quad$ where $0 \leq x \leq \pi/2$. Then,

(a) $A > 1$

(b) $A < 1$

(c) $0 < A \leq 1$

(d) $0 < A < 1$

Answer. (c)

Solution.

Note that $\sin^8 x \leq \sin^2 x$ and $\cos^8 x \leq \cos^2 x$. Hence, (c) is correct.

Problem.

$$\frac{x}{\cos a} = \frac{y}{\cos\left(a - \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(a + \frac{2\pi}{3}\right)}$$

Then, $x + y + z$ is

(a) 1

(b) 0

(c) -1

(d) None

Answer. (b)

Solution.

$$\begin{aligned} x + y + z &= k\left(\cos a + \cos\left(a - \frac{2\pi}{3}\right) + \cos\left(a + \frac{2\pi}{3}\right)\right) \\ &= k\left(\cos a + 2\cos a \cos\frac{2\pi}{3}\right) \\ &= k\cos a\left(1 + 2\cos\frac{2\pi}{3}\right) \\ &= 0 \end{aligned}$$

**Problem.** The value of

$$\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}$$

is

$(a)\ \frac{1}{4} \quad (b)\ \frac{1}{8} \quad (c)\ \frac{1}{2} \quad (d)\ none$

**Answer.** (b)

**Solution.**

$$\begin{aligned}
\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7} &= \cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\left(2\pi-\frac{8\pi}{7}\right)\\
&= \cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{8\pi}{7}\\
&= \frac{1}{8}\frac{\sin\frac{16\pi}{7}}{\sin\frac{2\pi}{7}}\\
&= \frac{1}{8}\frac{\sin\frac{14\pi+2\pi}{7}}{\sin\frac{2\pi}{7}}\\
&= \frac{1}{8}
\end{aligned}$$

**Problem.** Let $0 < x < \pi/6$, if $\sin x + \cos x = \sqrt{3/2}$. Then, $\tan x$ is

(a) $\frac{2+\sqrt{3}}{2}$

(b) $\frac{2}{2-\sqrt{3}}$

(c) $2 - \sqrt{3}$

(d) data is insufficient.

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**Answer.** $2 - \sqrt{3}$

**Solution.** We have $\sqrt{2}\sin(x + \pi/4) = \sqrt{3/2}$. Hence, $x = \pi/3 - \pi/4 = \pi/12$ or $x = 15$ degrees.

We know that $\cos 30 = \frac{1-\tan^2 15}{1+\tan^2 15}$

Solving for $\tan 15$ gives $2 - \sqrt{3}$.

Problem. The intervals where $\sin x - \cos x$ is negative is

(a) $(-\pi/4, 0) \cup (\pi/2, 3\pi/4)$

(b) $(\pi/4, 5\pi/4)$

(c) $[0, \pi/4) \cup (5\pi/4, 2\pi)$

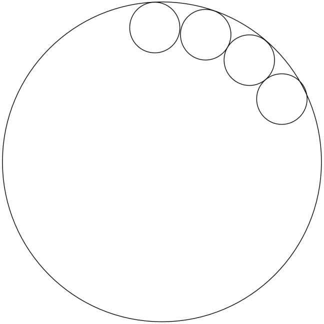
(d) None

Answer. (c)

Solution. $\sin x - \cos x = \sqrt{2}\sin(x - \pi/4) < 0 \Rightarrow \pi < x - \pi/4 < 2\pi$
$\Rightarrow 5\pi/4 < x < 9\pi/4$

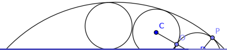
**Problem.** Let $n \geq 3$ smaller circles touch their neighboring smaller circles and the big circle as shown in the picture. If $r$ is the radius of a smaller circle, the radius of bigger circle is

$(a)\, r \csc\left(\frac{\pi}{n}\right)$ $(b)\, r(1+\csc\left(\frac{2\pi}{n}\right))$ $(c)\, r(1+\csc\left(\frac{\pi}{n}\right))$ $(d)$ none



**Answer.** (c)

**Solution.**



Problem.

$$\cos 2x = \sqrt{2}(\cos x - \sin x)$$

Then, $\tan x$ is

(a) $\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 1$

(b) 1

(c) 1, -1

(d) None

Answer. (b)

Solution. $\cos 2x = \sqrt{2}(\cos x - \sin x)$
$\Rightarrow \cos^2 x - \sin^2 x = \sqrt{2}(\cos x - \sin x)$
$\Rightarrow \cos x + \sin x = \sqrt{2}$ assuming $\cos x - \sin x \neq 0$
$\Rightarrow x + \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi$ for some integer $k$
$\Rightarrow x = \frac{\pi}{4} + 2k\pi$ for some integer $k$

Hence, $\tan x = 1$. If $\sin x = \cos x$, we get $x = \frac{\pi}{4} + k\pi$ for some integer $k$, again giving us the same solution.

Problem. The equation

$$\cos x = y + \frac{p}{y}$$

has a real solution for $x$ if

(a) $p = 1/2$

(b) $p \geq 1/4$

(c) $p \leq 1/4$

(d) none

Answer. (c)

Solution. $y^2 - y\cos x + p = 0$ has real solution if $\cos^2 x - 4p \geq 0$.This means $p \leq \frac{\cos^2 x}{4}$. Hence, $p$ cannot be greater than $\frac{1}{4}$.

Problem.

$$\tan x = \frac{1+\sqrt{1-p}}{1+\sqrt{1+p}}$$

Then $\cos 8x$ is (a) $2p^2-1$ (b) $-2p\sqrt{1-p^2}$ (c) $2p^2+p$ (d) None

Answer. (a)

Solution. Put $p = \cos y$. Then,

$$\begin{aligned}\tan x &= \frac{1+\sqrt{1-\cos y}}{1+\sqrt{1+\cos y}} \\ &= \frac{1+\sqrt{2}\sin y/2}{1+\sqrt{2}\cos y/2} \\ &= \frac{\cos \pi/4 + \sin y/2}{\sin \pi/4 + \cos y/2} \\ &= \tan\left(\frac{\pi}{8}+\frac{y}{4}\right)\end{aligned}$$

$\Rightarrow x = \frac{\pi+2y}{8} \Rightarrow 2y = 8x-\pi \Rightarrow \cos(2y) = \cos(8x) = 2\cos^2 y - 1 = 2p^2-1$

Problem. If $\sin a = p, \mid p \mid < 1$, then the quadratic equation whose roots are $\tan\left(\frac{a}{2}\right)$ and $\cot\left(\frac{a}{2}\right)$ are

(a) $px^2 + 2x + p = 0$

(b) $px^2 - x + p = 0$

(c) $px^2 - 2x + p = 0$

(d) None

Answer. (c)

Solution. Note that product of the roots is 1.
Sum of the roots,

$$\tan\left(\frac{a}{2}\right) + \cot\left(\frac{a}{2}\right) = \frac{\tan^2\left(\frac{a}{2}\right) + 1}{\tan\left(\frac{a}{2}\right)} = \frac{\sec^2\left(\frac{a}{2}\right)}{\tan\left(\frac{a}{2}\right)} = \frac{1}{\sin\left(\frac{a}{2}\right)\cos\left(\frac{a}{2}\right)} = \frac{2}{\sin a}$$

Now form the equation

$$x^2 - (\text{sum of the roots})x + (\text{product of the roots}) = 0$$

to get (c)

**Problem.** The set of values of $a$ such that $\sec^2 x + \tan x = a$ holds for some $x$ is

(a) $[0, 3/4]$

(b) $(1/2, \infty)$

(c) $(-\infty, 3/4]$

(d) None

**Answer.** (d)

**Solution.** We have, $\tan^2 x + \tan x + 1 - a = 0$ This equation has a solution if the discriminant is non-negative.

Hence, $1^2 - 4(1-a) \geq 0 \ \Rightarrow a \geq \frac{3}{4}$

**Problem.** If $\tan a + \tan b + \tan c = \tan a \tan b \tan c$ where none of the tan values are unbounded. Then, which of the options is the most generic

(a) a, b, c must be angles of a triangle.

(b) sum of some two angles must be equal to the third

(c) a+b+c is an integral multiple of $\pi$

(d) None

**Answer.** (c)

**Solution.**

$$\begin{aligned} a+b &= k\pi - c \qquad \text{where k is an integer} \\ \tan(a+b) &= \tan(k\pi - c) \\ \frac{\tan a + \tan b}{1 - \tan a \tan b} &= -\tan c \\ \tan a + \tan b + \tan c &= \tan a \tan b \tan c \end{aligned}$$

**Problem.** $x = \tan a \neq 0, y = \tan 2a \neq 0$ and $\tan a + \tan 2a = \tan 3a$

(a) $x = y$

(b) $xy = 1$

(c) $x + y = 0$

(d) None

**Answer.** (c)

**Solution.** We have

$$\begin{aligned} \frac{\tan a + \tan 2a}{1 - \tan a \tan 2a} &= \tan 3a \\ \tan a + \tan 2a &= \tan 3a(1 - \tan a \tan 2a) \\ \tan 3a - \tan 2a - \tan a &= \tan 3a \tan 2a \tan a \\ \Rightarrow \tan 3a \tan 2a \tan a &= 0 \\ \Rightarrow \tan 3a &= 0 \quad (\text{since } x \neq 0, y \neq 0) \\ \Rightarrow x + y &= 0 \end{aligned}$$

Problem. $1 + \cos(x - y) = 0$. Then

(a) $\cos x + \cos y = 0$

(b) $\sin x + \sin y = 0$

(c) $\sin x - \sin y = 0$

(d) none

Answer. (b)

Solution. $\Rightarrow \cos(x - y) = \pi + 2k\pi$ for some integer $k$.

$\Rightarrow x = y + \pi + 2k\pi$

$\Rightarrow \sin x = -\sin y$ and

$\cos y = \cos x$

Problem. 16. The number of real solutions of the equation $\sin(e^x) = 2^x + 2^{-x}$ is

$(a) 0 \quad (b) 1 \quad (c) 2 \quad (d)$ none

Answer. (a)

Solution. Note that RHS$= \frac{2^{2x}+1}{2^x} \geq 2$. Hence, the equation has no solution.

Problem. If $\cos 2x + 2\cos x = 1$. Then, $\sin^2 x(2 - \cos^2 x)$ is

(a) $\sqrt{5}$

(b) $\sqrt{-5}$

(c) -1

(d) 1

Answer. (d)

Solution. $2\cos^2 x - 1 + 2\cos x = 1$

$\Rightarrow \cos^2 x + \cos x - 1 = 0$

$\Rightarrow \cos x = \frac{-1+\sqrt{5}}{2}$

Substituting in $\sin^2 x(2 - \cos^2 x)$ gives 1.

Problem. The maximum and value of the function $f(x) =| min\{\sin x, \cos x\} \mid$ is

(a) The is no maximum value.

(b) 1

(c) $1/\sqrt{2}$

(d) 1/2

Answer. (c)

Solution. Note that $\sin x$ and $\cos x$ are equal when they are equal to $1/\sqrt{2}$. And that gives the maximum values.

Problem. In how many ways can you distribute 11 identical balls in 10 different boxes so that no box is empty?

$(a)\, 10 \quad (b)\, 11 \quad (c)\, 11^{10} \quad (d) 10^{11}$

Answer. (a)

Solution. Distribute 10 balls in 10 boxes. The last one has 10 choices.