lecture-3 solutions

1. Find $\cot(-855^\circ)$

Sol $\cot(-855^\circ) = -\cot(855^\circ)$

Note that $855^\circ = (360 \times 2)^\circ + 135^\circ$

$$\Rightarrow -\cot(855^\circ) = -\cot(135^\circ)$$
$$= -\cot(90+45^\circ)$$
$$= -(-\cot 45^\circ)$$
$$= \cot 45^\circ$$
$$= 1$$

2. Simplify $\sin(-480°)\sec(-390°) + \cot(315°)\csc(570°)$

$= -\sin(480°)\sec(390)° + \cot(315°)\csc(570°)$

$= -\sin(360+120)°\sec(360+30)°$

$+ \cot(360-45)°\csc(360+210)°$

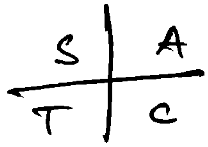
$= -\sin 120°\sec 30° + \cot(-45°)\csc(210°)$

$= -\dfrac{\sin(90+30)}{\cos 30} + \dfrac{-\cot(45)}{\sin(180+30)}$

$= \dfrac{-\sin 60°}{\cos 30°} + \dfrac{-1}{-\sin 30°}$

$= -1 + \dfrac{+1}{1/2} = 1$

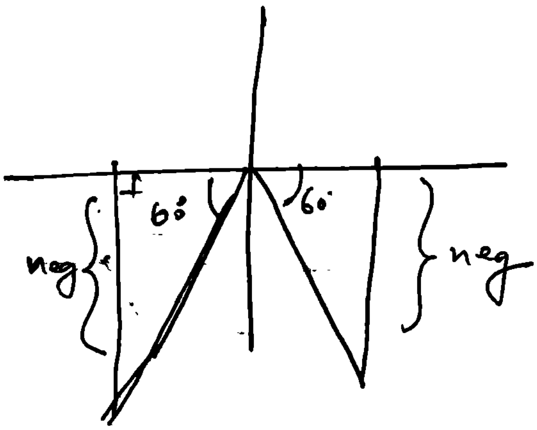
3. Find θ when $\sin\theta = -\sqrt{3}/2$

Sol Note that by 

rule,

θ can lie in third or fourth quadrant only. By checking, we find that

$\theta = 180 + 60 = 240^\circ$ or $\theta = 360 - 60 = 300^\circ$



4. Prove

$$\cos 100^\circ \cos 10^\circ + \sin 100^\circ \sin 10^\circ = 0$$

Sol: We would turn the trig ratios with angles greater than 90° into trig ratios with angle less than 90°.

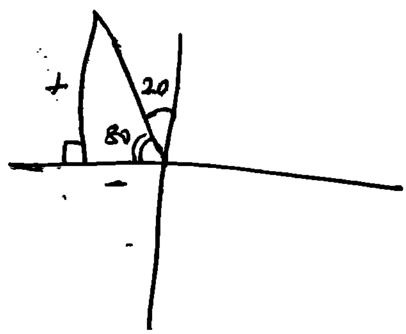
$$\cos 100^\circ \cos 10^\circ + \sin 100^\circ \sin 10^\circ$$

$$= \cos(90+20)^\circ \cos 10^\circ + \sin(90+20)^\circ \sin 10^\circ$$

$$= -\cos 80^\circ \cos 10^\circ + \sin 80^\circ \sin 10^\circ$$

$$= -\cos(90-10)\cos(90-80) + \sin 80 \sin 10$$

Using complementary angle property,



$$= -\sin 10^\circ \sin 80^\circ + \sin 80^\circ \sin 10^\circ$$

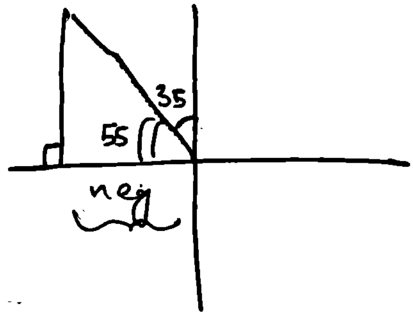
$$= 0$$

Evaluate :

5. $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ$

$+ \cos 300^\circ$.

Note that



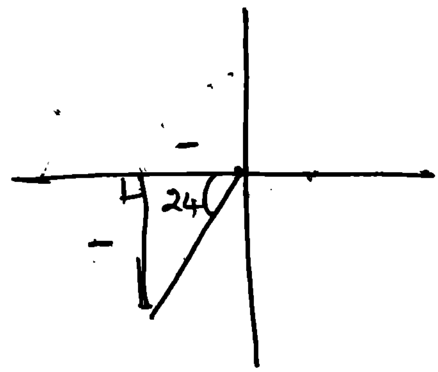
$\cancel{\cos 24^\circ}$

$$\cos 125^\circ = \cos(90+35)^\circ$$

$$= -\cos(55^\circ)$$

$$\cos(204^\circ) = \cos(180+24)$$

$$= -\cos(24^\circ)$$



The expression reduces to,

$$\cos(300^\circ) = \cos(360-60)^\circ$$

$$= \cos(-60^\circ) \quad \text{as} \quad \cos(-\theta) = \cos\theta$$

$$= \cos(60^\circ)$$

$$= 1/2$$

6. Let A, B, C be the angles of a Δle.

Then, prove

$$\sin(A+B) = \sin C$$

$$\cos(A+B) = -\cos C$$

$$\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$$

$$\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$$

Sol. $\sin(A+B) = \sin(\pi - C)$ [as sum of the angles of a Δle is π]

$= -\sin C$

$|||^{ly}$ $\cos(A+B) = \cos(\pi - C)$

$= -\cos C$

$\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{\pi - C}{2}\right)$

$= \sin\left(\pi/2 - \frac{C}{2}\right)$

By complementary angle property,

$= \cos(C/2)$

$|||^{ly}$ $\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{\pi - C}{2}\right)$

$= \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$= \sin(C/2)$

7. Let ABCDA be a cyclic quadrilateral. Then prove,

a. $\sin A + \sin B - \sin C - \sin D = 0$

b. $\sum \cos A = 0$

c. $\sum \tan A = 0$

d. $\sin(A+B) + \sin(C+D) = 0$

Sol. Let ABCDA be a cyclic quadrilateral.

Then. $\angle A + \angle C = \pi$

$\angle B + \angle D - \pi$

We shall A inplace of $\angle A$.

Consider,

$\sin A + \sin B = \sin(\pi - C) + \sin(\pi - D)$

$= +\sin C + \sin(+D)$

This proves (a).

Consider,

$\cos A + \cos B = \cos(\pi - C) + \cos(\pi - D)$

$= -\cos C + (-\cos D)$

This prove (b).

Similarly,

$$\tan A + \tan B = \tan(\pi - C) + \tan(\pi - D)$$
$$= -\tan C + (-\tan D)$$

This proves (c).

Consider,

$$\sin(A+B) = \sin[(\pi - C) + (\pi - D)]$$
$$= \sin[2\pi - (C+D)]$$
$$= +\sin[-(C+D)]$$
$$= -\sin(C+D)$$

proves (d). But $\sin(A+B) = \sin[2\pi - (C+D)]$ as angles sum upto 2π. Hence, (d) holds for any convex quadrilateral.