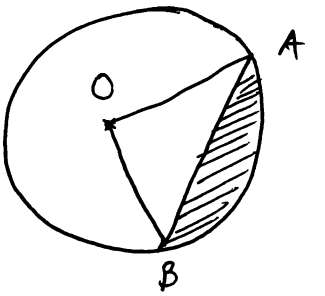
Problems week 1

1. Let AB be a chord of length 1 of a circle of unit radius (ie., circle of radius 1). Find the shaded area.



Solution: Let O be the centre of the circle and let AB be the chord of length 1.

Then, Δle OAB is equilateral.

$\Rightarrow \angle AOB = \pi/3$

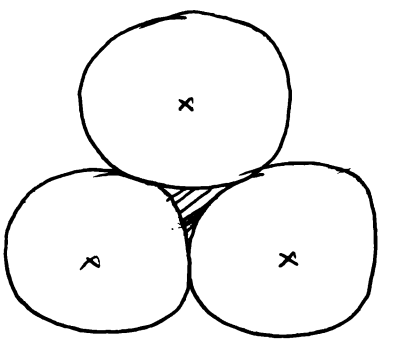
Area of the shaded part

= Area of Sector OAB − Area of ΔOAB

$$= \frac{1}{2} \cdot (1)^2 \cdot \pi/3 - \frac{\sqrt{3}}{4} \cdot (1)^2$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{4}$$

2. Find the shaded area where all three circles of radius 1 touch each other.

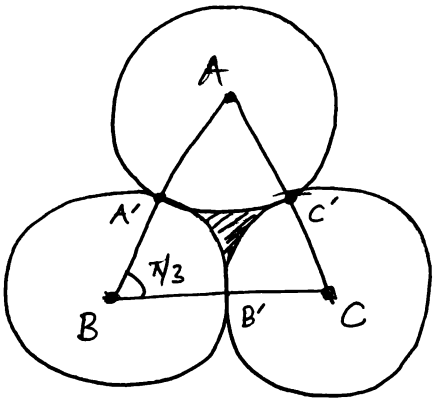


Solution:

Shaded area

= Area of Δle ABC

$-$ 3 (Area of sector $BA'B'$)



Note that $AB = 2$ (twice the radius)

Hence, $AC = BC = 2$ Hence, ΔABC is equilateral.

Area of Δle $ABC = \frac{\sqrt{3}}{4} 2^2 = \sqrt{3}$

Note: Area of sector $BA'B'$ = Area of sector $CB'C'$

= Area of sector $AA'C'$

(by symmetry)

Area of sector $BA'B = \frac{1}{2} \times 1^2 \times \pi/3 = \frac{\pi}{6}$

Shaded area $= \sqrt{3} - 3\left(\frac{\pi}{6}\right) = \sqrt{3} - \pi/2$

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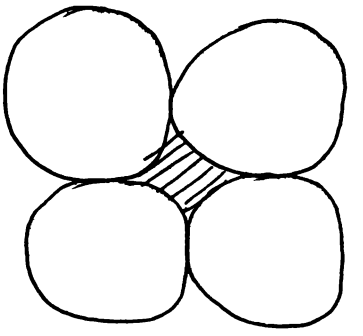
3. Find the shaded area where all circles have radius 1.

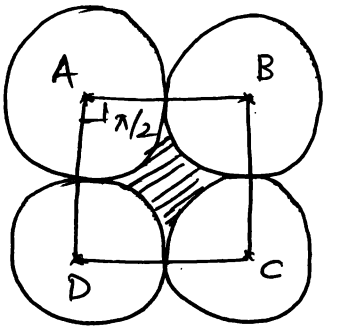


Solution:

Shaded area

= Area of the square ABCD

− Area of the four sectors.



$$= 2^2 - 4\left(\frac{1}{2} \cdot 1^2 \cdot \frac{\pi}{2}\right)$$

$$= 4 - 4 \cdot \frac{\pi}{4} = 4 - \pi$$

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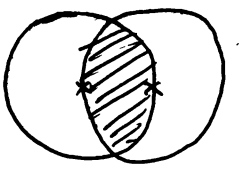
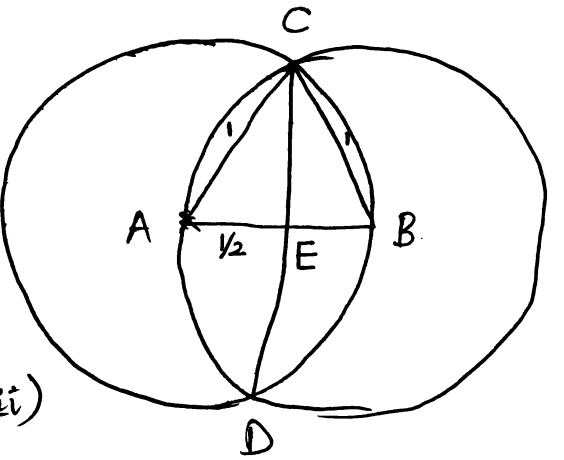
4. Find the shaded area where both circles have radius 1 and they pass through each other's centre.



Solution:

Note the shaded area

= 2 (Area of the sector ACBD − Area of the ΔACD)



$AC = CB = AB$ (all are radii)

$\Rightarrow \angle CAE = \pi/3 \quad (60^\circ)$

$\Rightarrow \angle CAD = 2\pi/3 \quad (120^\circ)$ ... by symmetry

By pythagoras theorem,

$$CE = \sqrt{AC^2 - AE^2}$$

$$= \sqrt{1^2 - (1/2)^2}$$

$$= \frac{\sqrt{3}}{2}$$

$$\Rightarrow CD = \sqrt{3} \quad \text{--- (by symmetry)}$$

Area of the sector ABCDE $= \frac{1}{2} \times 1^2 \times \frac{2\pi}{3}$

$$= \pi/3$$

Area of $\Delta ACD = \frac{1}{2} \times CD \times AE$

$$= \frac{1}{2} \times \sqrt{3} \times 1/2$$

$$= \frac{\sqrt{3}}{4}$$

Shaded area $= 2\left(\pi/3 - \sqrt{3}/4\right)$

(end)